Tommaso Virgilii, why

It is a simple argument, but people keep making it complicated, this is a simple explanations.

1 Transmission lines theory, intro

Usually a circuit is treated in the approximation that there is only a time t equal for all parts in the circuit. This means that everything has 0 propagation time, a change in a part of the circuit is instantaneously perceived in all the circuit. This means that the circuit is smaller than the characteristic length of the process, that is given by the velocity of the propagation of the electric signal that is circa c multiplied by the tipycal time of the process,, $\tau = 1ns$ for GHz application, this gives a $\lambda = 30cm$, so a circuit of $3cm$ can be treated with the KVL and KCL. But there are case of big circuits indeed, like transmission lines. These are long coupled wires, like in the figure down below. We only consider the inductance and the capacitance per unit length[1] because they

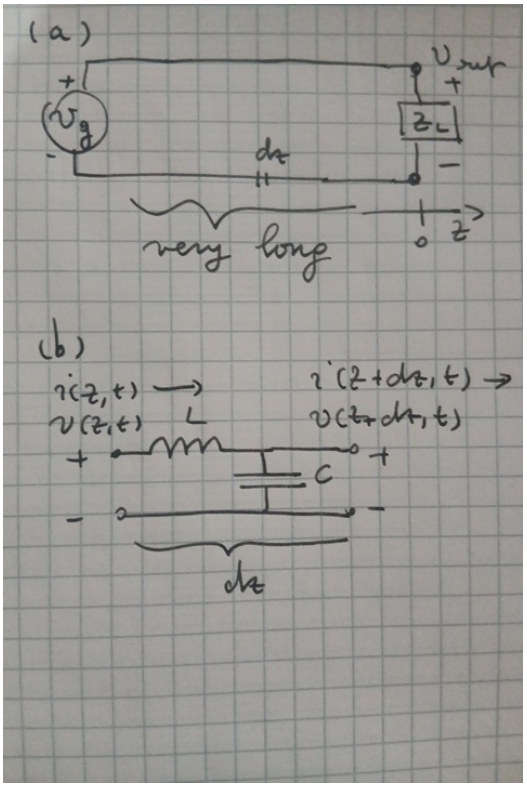


Figure 1: Lossless transmission lines. (a) general scheme, (b) infinitesimal element of length dz.

are the dominant effect when dealing with lines (resistance make losses that we, physicist usually ignore). So now the problem has been made local, i.e. this small circuit of length dz is smaller than the λ of the circuit surely.

[1] It is easy to put in the losses, by adding resistance in series and in parallel to the part of the line, this is not educational tho. The losses add an exponential decay (you guess...).

So there are two nodes in this circuit, so we need two equation, we could use KVL on the external loop and get in Fourier transform:

$$\frac{dv(z)}{dz} + j\omega L i(z) = 0. \tag{1}$$

And KCL applied to the upper node gives

$$\frac{di(z)}{dz} + j\omega C v(z) = 0 \tag{2}$$

That deriving 1 and plugging into the other gives:

$$\frac{d^2v(z)}{dz^2} - \omega^2 LC v(z) = 0$$

and doing the opposite

$$\frac{d^2i(z)}{dz^2} - \omega^2 LC i(z) = 0$$

Defining now $\gamma^2 = \omega^2 LC$ we get solutions for v, i of this kind:

$$v(z) = v^+ e^{-j\gamma z} + v^- e^{j\gamma z} \tag{3}$$

That in this convention of considering the temporal phase equal to $e^{j\omega t}$ are the propagating and anti-propagating waves. The current is obtained by eq. 1:

$$i(z) = (v^+ e^{-j\gamma z} - v^- e^{j\gamma z})\sqrt{\frac{C}{L}} \tag{4}$$

Now it is natural to define $Z_0 = \sqrt{\frac{L}{C}}$. Now the coefficients v^+, v^- are determined by the conditions of the generator and of the load. Putting $z = 0$ at the load Z_L we can work out what happens to it.

$$v(0) = v^+ + v^-$$
$$i(0)Z_0 = v^+ - v^-$$

Where for our conventions $v(0)$ is the voltage at the ends and $i(0)$ is the current passing in Z_L. Now we write the definition: $Z_L = v(0)/i(0)$. Thus dividing the two precedent equation one by the other and substituting the definition we ger:

$$v^- = \frac{Z_L - Z_0}{Z_L + Z_0} v^+, \tag{5}$$

that is one of the two boundary conditions, (the other being the generator). Usually one notes out that if $Z_L = Z_0$ there is no reflected (anti-propagating) wave.

2 Lossy line

This is a lossy line dz element, solve it as an exercise, show how the velocity of the wave is ω dependent:

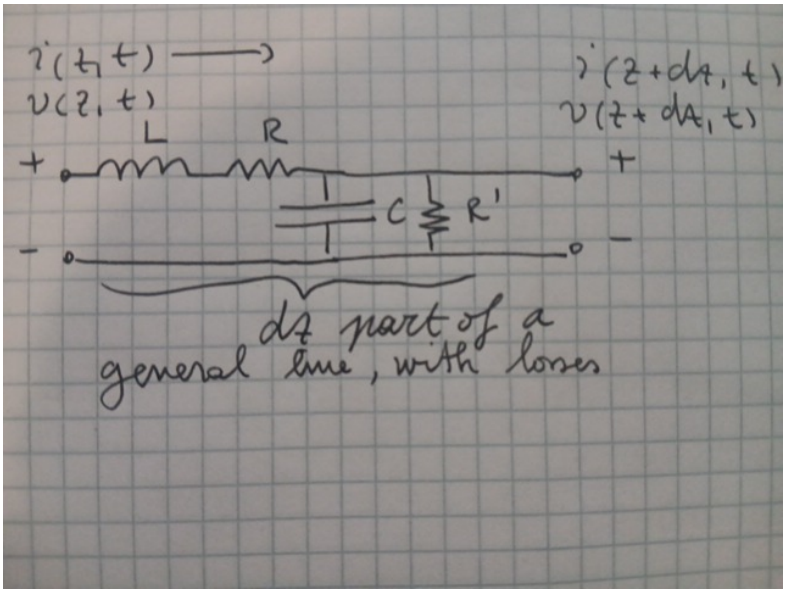


Figure 2: Lossy line infinitesimal element